A Sticky Gum Problem

This extended problem starts with some specific problems, and then asks you to generalize what you've learned from them.

Here's the first problem:

1. Ms. Hernandez comes across a gumball machine one day when she is out with her twins. Of course, the twins each want a gumball. What's more, they insist on being given gumballs of the same color. (They don't care what color the gumballs are, as long as they're the same color.) Ms. Hernandez can see that there are only white gumballs and red gum balls in the machine. The gumballs are a penny each, and there is no way to tell which color will come out next. Ms. Hernandez decides she will keep putting in pennies until she gets two gumballs of the same color. Why is three cents the most she might have to spend in order to satisfy her twins?

2. The next day, Ms. Hernandez passes a different gumball machine. This one has three colors – red, white, and blue. What is the most Ms. Hernandez might have to spend at this new gumball machine in order to get matching gumballs for her twins.

3. Here comes Mr. Hodges with his triplets past the three-color gumball machine described in question two. Of course, his children also insist that they all get the same color gumball. What is the most Mr. Hodges might have to spend?

After you have answered the question above, create some examples of your own. You may want to begin with more examples about the Hernandez twins, using different numbers of colors. Or you may want to create other examples using the three- color gumball machine and larger sets of children.

As you create and solve examples of your own, look for a way to organize the information and look for patterns. Your ultimate goal is to find a formula so that, if someone tells you the number of colors and the number of children, your formula will tell you the maximum that the parent might need to spend.

The gambler's Fallacy

Introduction-

In the game of roulette, a ball is spun around in a roulette wheel, and it lands either in a red slot or in a black slot (There's also a very small chance that it will land in a green slot, but in this problem, we will simplify things by ignoring that fact.) The chance of the ball landing in a red slot is the same as its chance of landing in a black slot.
Some gambler's use the following strategy for winning at roulette: They watch a wheel and if it gets a certain number of reds in a row, they bet on black, since they figure it black's turn. Similarly, if they see a string of blacks coming up, they bet on red, since they figure red will be more likely than black after a string of blacks.

The experiment:

Do this experiment with a partner.

Flip a coin 25 times, and record each flip as heads (H) or tails (T), according to the outcome.

When you have completed all 25 flips, you will get a list of 25 letters, made up of H's and T's. Now start from the beginning of this list and find the first instance of three flips in a row that are identical (either three heads in a row of three tails in a row). We will call three identical flips in a row a triplet.

Record whether the flip that followed this first triplet was the same as the letters in the triplet or different. Then move to the next triplet and again record whether the flip that followed it was the same as, or different from, the letters in the triplet. Continue in this way through your whole list. (Note: ignore the last triplet at the end of your 25 flips, as nothing follows it) Then find out how many "same's" and how many "different's" you got.
If you have four identical flips in a row that give you two triplets. For example, suppose you have HHHHT as part of your record. As shown below, the fist three H's form a triplet that is followed by an H (a same): the second, third and fourth H's also form a triplet, followed by a T.

Coincidence or Causation?

In each of the following situations, you are to decide whether you think the past will or will not have a certain influence on the future?

1. A baseball player has averaged hitting a home run once every seven games for most of the season. She has just his a home run in each of the last three games.

2. It seems to Mr. Bryant that every time he comes along Pine Street to the traffic light at the intersection with Lincoln Ave, the light is red. He is so infuriated with this situation that he contacts the city planner. The city planner reports that the light is set so that the cars on Pine St. and the cars on Lincoln Ave are given equal time to pass through the intersection. If you are driving right behind Mr. Bryant one morning and come to that traffic light, do you think that your chances of getting a red light are greater than, less than, or equal to those given by the city planner?

3. The Happy Days Ice Cream Cone Company claims that, on average, only about 1 out of every 100 boxes of their famous ice cream cones will contain a broken cone. The company gladly replaces any box containing a broken cone. You go to the store and purchase a box of Happy Days Ice Cream Cones. Upon arrival at home, you discover that one of the cones is broken. Feeling somewhat cheated, you return the box to the place of purchase and exchange it for a new box. Just to be sure, you immediately check the new box for broken cones. Is the chance that the new box contains a broken cone different from 1 out of 100?

Paula's Pizza

Paula's favorite pizza place offers six toppings – sausage, onions, mushrooms, pepperoni, olives, and peppers. Paula ordered a pizza with mushrooms and olives.

Unfortunately, the server only wrote down that Paula ordered two toppings, and didn't write down which two they were. The chef doesn't know Paula, and decided to pick two toppings at random.

1. How many different two- topping pizzas are possible altogether?
2. What is the probability that Paula will get the pizza that she ordered? What is the probability that she'll get something different?
3. Paula actually likes all of the toppings except sausage and pepperoni. What is the probability that she will get a pizza she likes? What is the probability that she'll get a pizza she doesn't like?

Waiting for a Double

In many games that use dice, such a backgammon, you roll two dice at a time. Often special rules apply when you roll a double. (A double means having the same number show on both dice). So you might want to know how long it takes to get a double. You record the number of rolls it took to get a double.

Example:

First Roll	Dice come up 3 and 4.
Second Roll	Dice come up 2 and 5.
Third Roll	Dice come up 5 and 3.
Fourth Roll	Dice come up 2 and 2.

It took four rolls to get a double.

1. Predict the average number of rolls it will take to get a double. Write a sentence or two explaining why you made that prediction.
2. Do the experiment ten times. That is, for each experiment, roll a pair of dice until you get a double, counting how many rolls it takes. Write down the number of rolls needed each time.
3. Use the data you gathered in question 2 to answer these questions.
   a. What is the largest number of rolls it took to get a double? What was the smallest?
   b. What was the average of the ten experiments?
4. How close is the average you found in question 3b to the prediction you made in question 1? Would you revise your prediction new, based on your experiments? Why or why not?

0 or 1, or never to always

For each of the probabilities below, think up two situations that have the given probability.

In one of those two situations, the probability should be based on a theoretical model. In the other situation, the probability should be based on observed results. (you can be imaginative about this)

1. Probability = 0
2. Probability = 2/7
3. Probability = 75%
4. Probability = 1
5. Probability = 2.3
6. Probability = 0.1

What's on Back?

In a certain game there are three cards.

- One card has an X on both sides.
- One card has an O on both sides.
- One card has an X on one side, and an O on the other side.

The three cards are placed in a bag, and the bag is shaken. You draw out one card and look at one side only. You cannot look at the other side or at the other cards. The goal is to predict whether there is an X or O on the other side of the card you drew out.

The are many strategies for doing this, some good, some not as good. Here are two possible strategies.

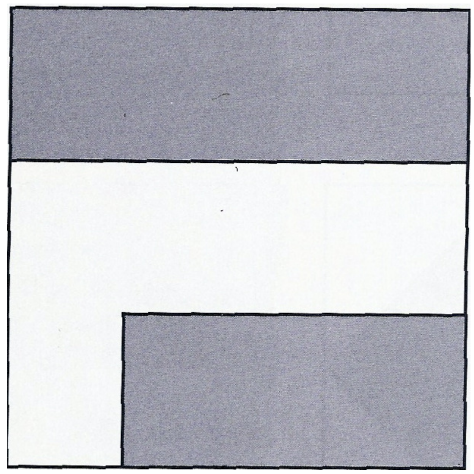
- Predict that the mark on the other side will be different from the mark you see. (that is, if you see X, predict O; and if you see O, predict X)
- Always predict that the mark will be an X.

No strategy will be successful all of the time, so you should try to find the probability of success for each strategy you consider. Your ultimate goal is to find the strategy that has the highest possible probability of being right.

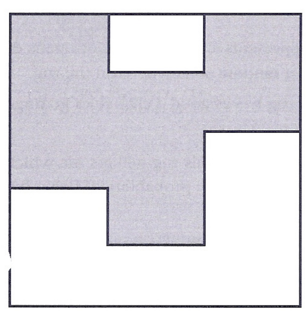
Bingo games

Imagine that each diagram in this activity represents a bingo board. A trap door opens directly over the board and a dart falls down, landing at random somewhere on the board. "At random" means that every point on the rug has a as good of a chance of getting a hit as every other point.

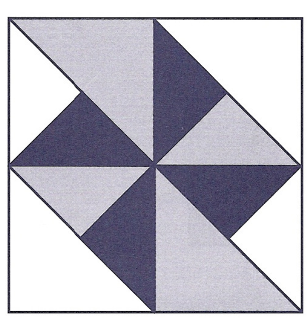
1. If you were trying to predict which part of this rug will get hit, which color would you choose, gray or white? What is the probability of being hit for each color?



2. For each of the rugs below, decide which color you would predict at most likely to be hit. For each color, find the probability of being hit.

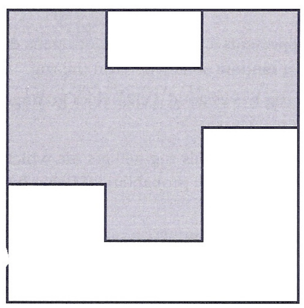


A



Pointed Rugs

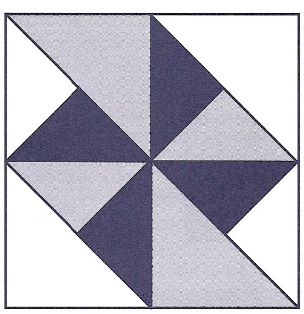
In Rug Games, you decided which color was most likely to be hit by a falling dart for each of the rugs below. In this assignment you are asked to work again with these rugs. But in this assignment, points are awarded for each color. This means that your choice of color involves more than just finding probabilities- you must also take into account the number of points that are awarded each time that the dart lands on a certain color.
For each rug, decide which color is the best to bet on to maximize your points in the long run. (hint: Imagine dropping a dart a large number of times, and decide which color would likely to give you the most points.)



Rug A

Gray: 6 points
White: 8 points

A



Rug B

Gray: 5points
White: 6 points
Black: 10 points

Mia's Cards

Mia is playing a game that involves picking a card from a standard deck. A standard desk consists of 52 cards, 13 from each suit (which are clubs, spades, diamonds, hearts). In Mia's game, she mixes up the cards and then picks a card a random from the deck. She gets 10 points if it's a heart, and 5 points for any other suit. If she does this many times, what will her average number of points for each trial? Explain your answer.

The counters game

Each player in this games needs a board, which consists of 11 boxes numbered from 2 through 12, as shown below.

2	3	4	5	6	7	8	9	10	11	12

At the start of the game, players each place 11 counters on their individual boards. The counters can be placed in the different boxes in any way the player chooses (including putting more than one counter in a single box). During the game, a pair of dice is rolled repeatedly, and the numbers on the dice are added each time. Every player who has any counters in the square corresponding to sum of the dice removes *one counter* from that square. (Even is a player has more than one counter in that square, the player does nothing on that roll. The winner of the game is the first player whose counters are all removed. The challenge of the game is to initially place the counters so that they will be removed as quickly as possible.

Play one or two practice games in your group. Just guess about where to place the counters.

Now, think about where to place the counters. Write a sentence or two explaining what you think would be a good way to place them and why.

Play some games in your group, with each member of the group using their strategy.

Discuss different strategies used in your group, in preparation for a competition between all groups. Choose a single strategy to use in the competition, and state what this strategy is.

Coins, Coins, Coins

Bobby was working on a problem that asks for the probability of getting two heads if you flip the coin twice.
He said if you flip twice, there are exactly three possible outcomes- two heads, one head and one tail, and two tails- and so the probability is getting two heads is one-third.

1. Explain why he is wrong. Make your explanation as clear as possible, using diagrams where needed.
2. Imagine you have two pockets and that each pocket contains a penny, a nickel, and a dime.
   You reach in and remove one coin from each pocket, the penny the nickel and the dime are equally likely to be removed.
   a. What are the possible amounts you could get for the total of the two coins?
   b. What is the probability that your two coins will total exactly two cents?
   c. What is the probability for each of the outcomes in 2a?

Spinner Games

Al and Betty are playing a game with the spinner shown here. Each time the spinner comes up in the white area, Betty wins one dollar from Al. Each time the spinner comes up in the colored area, Al wins four dollars from Betty.

1. In the long run, which of the two players is more likely to be the winner in this game? Write down your prediction and explain your reasoning.
2. now play the game for 25 spins and write down what happens.
3. If Al and Betty play 100 games, how far ahead is the expected winner likely to be?

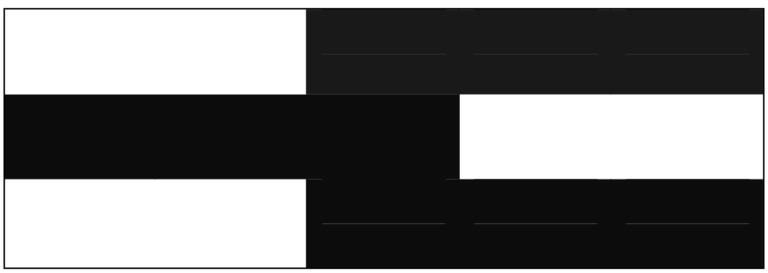
(Suggestion: You can make a spinner using a pencil and paper clip by bending open one end of the paper clip and then using the pencil to hold the other end in place as the paper clip spins.)

A fair rug game?

Tony and Crystal are sitting around a rug watching darts randomly fall from the ceiling. The rug they are using is pictured below. If the dart lands on the white part of the rug, Crystal wins $5 from Tony. If it lands on the black part, Tony wins $3 from Crystal.

Do you think this is a fair game? What is Tony's expected value for each turn? What's Crystal's?

If you think that the game is not fair to one of the players, change the amount of money they each win in order to make the game fair. (Don't change the rug.)



One- and- One

Sometimes in a basketball game, a player if presented with a situation known as a "one-and-one."

In a "one-and-one" situation, the player begins by taking a free throw. If the player misses, that's the end of it. But if the shot is successful, the player gets to take a second shot. One point is scored for each successful shot. So the player can end up with 0 points, 1 point, or 2 points.

Terry is a basketball player who has shown over a period of time that whenever she attempts a free throw, she has about a 60% probability of making it.

In a one-and-one situation, how many points is Terry most likely to score: 0,1, or 2? Write down your intuitive guess about the answer to this option.

A sixty percent solution

In this assignment, the situation is the same as the one described in the activity one-and-one.

Terry is in a one-and-one situation, and she a 60% probability of getting any given shot.

Devise some way to simulate the situation at home.

1. What was the most frequent outcome in your situation?
2. What was the average score per one-and-one situation?

Streak Shooting Kelly

When streak shooting Kelly steps up for a one-and-one situation, her chances of making the first shot are 80%. However, if she makes her first free throw, then there is a 90% chance that she will make her second free throw.

1. In what percentage of the situations will Kelly score no points? One point? Two points?
2. What is Kelly's expected value for each one-and-one?

Playing the Lottery

Many states raise funds through various lottery games.
Assume that each lottery ticket costs $1. The number of tickets sold and the value of a winning ticket often vary from week to week. Suppose that, for a certain week, about 14 million tickets were sold and that the winning ticket is worth $6 million.

1. Calculate the approximate expected value of a lottery ticket that week.
2. Do you think that buying a lottery ticket is a wise investment?

Martian Basketball

In Martian basketball, instead of having a one-and-one free throw situations, they have one-and-one-and-one situations.
In other words, if a player make both the first and second shots, then the player can take a third one as well (so the player can get 0 points, 1 point, two points, or three points). Suppose our friend Streak Shooting Shelly moved to Mars and played basketball there.
Shelly still shoots better when she just made a shot, but her overall quality is down because she is getting adjusted to the different gravity on Mars. So now she has a 60% probability of making her first shot. If she gets the first one, she has an 80% probability of making the second one, and if she gets the first two, her probability of getting the third is 90%.

1. How many points is she most likely to score in a one-and-one-and-one situation?
2. What is her expected value for each situation?

The Carrier's Payment Plan Quandary

In some places, newspapers are delivered by a newspaper carrier who was already paid for the newspapers. The carrier then collects from the customer, and keeps whatever he or she collects.
Suppose one day a customer says to the carrier, "Instead of collecting the usual $5 per week, how about if you just pick two bills at random out of this bag? You get to keep whatever you pick instead of the $5. If you choose to pick out of the bag, you'll do that every week from now on."

The customer shows the newspaper carrier the bag, which contains one $10 bill and five $1 bills. Thus, two sums are possible: $11 and $2. (the customer will replace the bills each week.)

1. Carry out a simulation with a reasonable amount of trials and decide which would be a better deal for the carrier.
2. Use the area model to compute the carrier's expected value for the alternative payment plan.
3. Which method do you trust more, simulation, or the theoretical model.

Bowls and Cones '0809

1) Jonathan delivers pizza several nights a week. Every night he works he gets a free pizza. He always brings home a 2-topping pizza with two different toppings of pineapple, sausage, mushrooms, onions, or anchovies. How many nights can he work without repeating a combination?

2) Jonathan's sister Johanna also works at the same pizza parlor and she always brings home a 3-topping pizza with three different toppings. If she likes the same 5 flavors as her brother, how many nights can she work without repeating a combination?

3) After Jonathan finishes delivering pizza, he always treats himself to a two-scoop bowl of ice cream at the ice cream shop next to the pizza parlor. If the ice cream shop has 24 flavors, how many different combinations of two scoops of ice cream can Jonathan create? (He always insists on getting two different flavors for his two scoops.)

4) Next, Johanna enters the scene. She likes her ice cream on a cone, and it's important to her which scoop is on top. After all, she says, eating chocolate and then vanilla is a different taste experience than eating vanilla and then chocolate. Like her brother, she always wants two different flavors. How many different two-scoop cones can Johanna make?

5) On an evening that Johanna wasn't working she asked Jonathan to pick her up a special three-scope cone on his way home from the pizza parlor. She wanted Chocolate, Vanilla, and Strawberry on her cone. Unfortunately she did not tell Jonathan the order she wanted. Not wanting to make her unhappy, he decided to get all possible cones with those three flavors. How many different cones would he have to get?

6) Jonathan realized he was lucky that Johanna had not wanted a 4-scoop cone. If she had, how many cones would he have had to buy?

7) After having that one 3-scoop cone, Johanna decided that she would not only eat 3-scoop cones from now on, but that she also wanted to try every possible 3-scoop combination. How many different cones could she get if the shop had 24 flavors and she always had three different flavors on each cone?

8) Suppose that Johanna wanted to try every possible 4-scoop cone. How many different cones could she get then?

9) Jonathan and Johanna are taking a road trip across the country this summer and decide that in every town they stop in they will sample the local ice cream shops.

10) In the first town they stop at Mookies Ice Cream Mart. They figure out that you can make 465 different 2-scoop bowls of ice cream. How many different 2-scoop cones can you make? (Hint: If there were only one possible 2-scoop bowl, how many cones could you make?)

11) In the next town they stop at Chilly Ice Cream Parlor where you can make 220 different 3-scoop bowls of ice cream. How many different 3-scoop ice cream cones can you make?

12) At the third town they stop at the Tasty Ice Cream Shop where you can make 210 different 4-scoop bowls of ice cream. How many different 4-scoop ice cream cones can you make?

13) At Tom's Frozen Foods, you can make 3024 different 4-scoop ice cream cones. How many different 4-scoop bowls can you make. (Careful: this one reverses the situation.)

14) Next they stop at Spud's Finest where you can make 792 different 5-scoop bowls of ice cream. How many different 5-scoop ice cream cones can you make?

15) Lastly they stopped at Ice Cream by Midge, where you can make 55440 different 5-scoop ice cream cones. How many different 5-scoop bowls can you make. (Careful: this one reverses the situation.)

16) In general how could you find the number of different cones of a particular size if you knew the number of bowl combinations?

17) In general how could you find the number of different bowls of a particular size if you knew the number of cone permutations?